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NONPARAMETRIC ESTIMATION OF SIGNALS MIXED WITH NOISE.(U)
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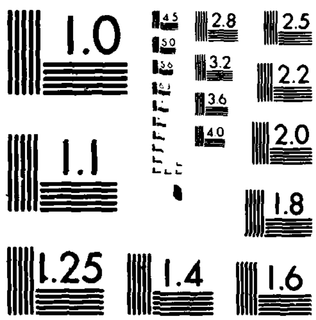

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR-TR- 82-0255
2. GOVT ACCESSION NO.: AD-A113 018
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): NONPARAMETRIC ESTIMATION OF SIGNALS MIXED WITH NOISE
5. TYPE OF REPORT & PERIOD COVERED: FINAL TECHNICAL REPORT Dec 1, 1980 - Sep 30, 1981
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): KAMAL C. CHANDA
8. CONTRACT OR GRANT NUMBER(s): AFOSR-81-0058

PERFORMING ORGANIZATION NAME AND ADDRESS: TEXAS TECH UNIVERSITY, DEPARTMENT OF MATHEMATICS, LUBBOCK, TEXAS 79409

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F, 2304/A5

CONTROLLING OFFICE NAME AND ADDRESS: DIRECTORATE OF MATHEMATICAL AND INFORMATION SCIENCES, AIR FORCE OFFICE OF SCIENTIFIC RESEARCH, BOLLING AIR FORCE BASE, WASHINGTON, D.C. 20332

12. REPORT DATE: February, 1982
13. NUMBER OF PAGES: Nine

MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):

15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION DOWNGRADING SCHEDULE

DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DTIC ELECTE MAR 0 6 1982 S D E

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Nonstationary time series, nonparametric statistics, linear processes, signals mixed with noise, chi-square goodness-of-fit tests, central limit theorem, sample quantiles, kernel-type density estimators.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Let $\{X_t, t = 0, \pm 1, \ldots\}$, $(X_t = S_t + W_t)$ be a stochastic process which represents a signal process $\{S_t\}$ mixed with noise $\{W_t\}$. It is assumed that $\{W_t\}$ is a linear process defined by $W_t = \sum_{r=0}^{\infty} g_r Z_{t-r}$ where $\{Z_t\}$ is a pure white noise, $\{g_r\}$ is a sequence of real numbers and the sum above is to be understood in some probabilistic sense. A set of data which consists of

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20. ABSTRACT (continued)

observations on X_j $(1 \leq j \leq n)$ is used to compute statistics of various types. These statistics are employed as tools of estimation and tests of hypotheses about $\{W_j\}$ and $\{S_t\}$ and the sampling properties of these statistics are investigated through appropriate versions of the central limit theorem and the strong law of large numbers.

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AFOSR-TR- 82-0255

Final

Nonparametric Estimation of Signals Mixed With Noise

AFOSR-81-0058

Kamal C. Chanda

SCIENTIFIC DESCRIPTION OF RESEARCH AND RESULTS

1. INTRODUCTION

Analysis of a set of evolutionary or nonstationary time series data is traditionally carried out by the use of regression and spectral methods. Although these procedures are, to some extent, nonparametric in nature, the basic assumption implicit in such data analysis has usually been that the time series is Gaussian or nearly so. We do not know very well how efficient and useful these procedures are in case the data structure is distinctly non-Gaussian.

Classical data analysts who handle data sets composed of independent observations have noticed over the last three decades that robust and adaptive nonparametric methods have worked very efficiently even for situations where large implicit variability in the data has effectively ruled out a Gaussian model. In recent years several concerted attempts have been made to adapt these nonparametric methods to the area of time series analysis with some success (for a partial listing see Kassam and Thomas [13] and Basawa and Prakasa Rao [1]).

The present Technical Report (along with the previous one communicated under AFOSR Contract # F 49620-79-C0194) is based on work done as an attempt to extend these methods of nonparametric statistics to selected areas of time series analysis.

2. MAIN PROBLEM

We have assumed throughout that we are dealing with a discrete time series $\{X_t, t \in J\}$ where J is the set of integers and that the given data set consists of observations on X_i $(1 \le i \le n)$. In general $\{X_t\}$ is nonstationary. But since nonstationarity in a random process can take infinitely varied forms, it seems reasonable to impose some restrictive conditions on such nonstationarity. We have accordingly assumed that

$$X_t = S_t + W_t \ , \ t \in J, \tag{1}$$

where $\{S_t\}$ is a 'signal' process which may be nonrandom and partially known or may itself be random and $\{W_t\}$ is stationary or a slowly evolving process. In the past, models of the type (1) have been investigated through standard regression and spectral analysis, but usually under the implicit assumption of a Gaussian model. Some recent attempts have been made to avoid this assumption and work within the framework of nonparametric methods which have proved useful and efficient for data analyses involving independent observations. One may mention in this context the works of Basawa and Prakasa Rao [1], Davisson, Feustal and Modestino [9], Gastwirth and Rubin [10] , Grossman [11], Kanefsky and Thomas [12], Kassam and Thomas [13], Koul [14] and Wolff, Gastwirth and Rubin [16] among others. In order to take advantage of the power of the standard nonparametric methods, and at the same time recognize the interdependence of the series of observations it may be reasonable to use a model which directly describes the probabilistic structure of the process instead of depicting its second order (spectral) properties. The advantage of using a somewhat explicit but general probabilistic model (which, in particular, may be Gaussian) is quite clear - we may be able to handle the data analysis in a comparatively more precise quantitative manner and at the same time assure ourselves that these tools have adequate robustness properties.

To this end we introduce the following probability model for $\{W_t\}$. We assume that $\{W_t\}$ is a linear process in the sense that

$$W_t = \sum_{r=0}^{\infty} g_r Z_{t-r} \tag{2}$$

where $\{Z_t\}$ is a pure white noise process and the infinite sum on the right hand side of (2) converges in some probabilistic sense. Observe that the classical mixture of the autoregressive moving average (ARMA) processes are special cases of (2). If we specify the distribution function (DF) F_0 of Z_1 we provide a complete probabilistic description of $\{W_t\}$. If F_0 is Gaussian $\{W_t\}$ is Gaussian. If F_0 is heavy-tailed, $\{W_t\}$ is very definitely non-Gaussian. Properties of such linear processes have been investigated in a series of articles (see Chanda [2],[3],[4],[5]).

3. METHODS OF INVESTIGATION

We have addressed ourselves mainly to the development of non-parametric tools of inference relating to a nonrandom signal $\{S_t\}$ mixed with noise $\{W_t\}$ resulting in the time series model (1). We have assumed that

$$S_t = \sum_{j=1}^{p} \alpha_j a_{tj} \quad t \in J, \tag{3}$$

where α_j $(1 \leq j \leq p)$ are unknown parameters and a_{tj} $(1 \leq j \leq p)$ are known constants. Then instead of using the conventional least squares estimates of α_j and hence of S_t we may use some other more efficient methods based on the likelihood function. This will entail adaptive estimation of α_j through estimation of the probability density function (PDF) f_0 of Z_1 or of the PDF f of W_1. We also consider the possibility of using sample quantiles for estimating S_t, and determine goodness-of-fit tests for identifying the distribution of $\{X_t\}$.

4. DESCRIPTION OF WORK COMPLETED

During the period July, 1979 - September, 1980 we have developed some simple analytic tools to handle the problems of inference relating to the model (1). These consist of several nonparametric statistics of which the sampling properties have been investigated in some details. We describe thse details in the sections 4.a and 4.b.

4.a. Chi-square Goodness-of-Fit Tests

We have assumed that $S_t = 0$ so that $X_t = W_t$ in (1). The DF F of X_1 will then involve unknown parameters in g_r ($r \geq 1$, $g_0 = 1$) and the DF F_0 of Z_1. In order to test simple and composite hypotheses about F we have considered the standard chi-square goodness-of-fit tests based on X_j ($1 \leq j \leq n$). Asymptotic sampling properties of these tests are then investigated under these null hypotheses. We conclude that these properties are largely determined by the multivariate probability structure of $\{X_t\}$ in (1). The results are given in Chanda [6] (a reprint of this article is enclosed with this Report).

4.b. Sampling Properties of a Class of Statistics

The class of statistics considered in 4.a. is somewhat generalized, although we continue to assume as in 4.a. that $X_t = W_t$. We define the vector process $\{\underset{\sim}{Y}_t\}$ where $\underset{\sim}{Y}_t = \underset{\sim}{h}\,(X_t, \ldots, X_{t+s-1})$

for some $s \geq 1$ and $\underset{\sim}{h}$ is a real vector function on R^s. The Central Limit Theorem and the Strong Laws of Large Numbers are then established for $\{\underset{\sim}{Y}_t\}$. These results are then used to establish asymptotic normality and consistency properties of a large class of statistics which provide parametric and nonparametric estimates for a class of signals mixed with noise described in model (1). The results have already been communicated in the Final Technical Report for AFOSR Contract #F - 49620-79-C-0194.

During the final phase of our investigation for the period December 1980 - September 1981 (under AFOSR Grant #81-0058) we have developed additional tools of inference for model (1) with $S_t = 0$. We have investigated the asymptotic sampling properties for a class of sample quantiles and considered the possibility of consistently estimating the PDF f of X_1. The details are described below in section 4.c. and 4.d.

4.c. <u>Sampling Properties for a Class of Sample Quantiles</u>

Let $\{X_t\}$ be defined as in (1) with $S_t = \theta$. In the past such a location parameter has been estimated reasonably well by quantiles Q_n computed from the sample. Conventionally $Q_n = X_{k_n:n}$, where $X_{i:n} \leq \ldots. \leq X_{n:n}$ are the order statistics for the sample with $k_n/n \to p$ as $n \to \infty$ where $0 < p < 1$. It is, however, interesting to find out what happens to Q_n when $k_n/n \to 0$ or 1, as $n \to \infty$. We have investigated the sampling properties of such a Q_n for the special case when $X_t = \theta + Z_t$. It is established that for various choices of α_n, β_n, and k_n (in particular when $k_n/n \to 0$ or 1 as $n \to \infty$ and under some regularity conditions $\mathcal{L}(Y_n) \to \mathcal{N}(0,1)$ as $n \to \infty$ where $Y_n = (Q_n + \alpha_n)/\beta_n$ $(\beta_n > 0)$. Under some additional conditions standard asymptotic expansion (in Edgeworth form) for the distribution of Y_n is derived. The results are given in Chanda [7] (a copy of the manuscript is enclosed with this Report).

4.d Estimation of the Density Function

Let $\{X_t\}$ be defined by (1) with $S_t = \theta$. As we have mentioned earlier we may estimate the location parameter adaptively by using some kind of likelihood function approach (see Stone [15]). This can be accomplished by using estimates of the PDF f_0 or f. Since, in general, it is difficult to set up the likelihood function of $X_1,\ldots,X_n$ in terms of f_0 we have attempted to use estimates of f instead. We have employed the conventional kernel-type density estimators and established that these statistics are strongly consistent and asymptotically normal. The results are given in Chanda [8] (a copy of the manuscript is enclosed with this article).

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